Matthew Tran
EE105

- LTI System
  - Linear - $L(\alpha x_1 + \beta x_2) = \alpha L(x_1) + \beta L(x_2)$
  - Time Invariant - $L(x(t-\tau)) = y(t-\tau)$

$$\sin\theta = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

$$\cos\theta = \frac{1}{2}(e^{j\theta} + e^{-j\theta})$$

  - Complex exponential $e^{j\pi}$ is eigenfunction of LTI
  - $H(\omega)$ is eigenvalue

$$L(e^{j\omega t}) = H(\omega)e^{j\omega t} = |H(\omega)|e^{j(\omega t + \angle H)}$$

  - $H(j\omega)$ for steady state, $H(s)$ for transient response
    - e^{st}
  - Dirac Delta - $\delta(t)$
    - $x(\tau) = \int \delta(t-\tau)x(t)dt$ // sifting prop
    - $x(t) = \int x(\tau)\delta(t-\tau)d\tau$
  - Impulse Response $h(t) = L(\delta(t))$
  - Convolution

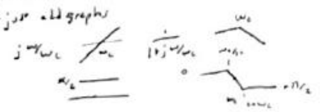
$$L(x(t)) = y(t) = h(t) * x(t)$$
$$= \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau$$

  - linearity super useful
  - Laplace Transform - $H(s) = \int_{-\infty}^{\infty} h(t)e^{-st}dt$
  - $Y(s) = H(s)X(s)$
  - Fourier Series - $x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f t}$
  - FT - $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
- AC Circuits / Analysis
  - transfer functions
  - impedance important
  - Trans Impedance Amplifier (TIA) - current to voltage
    - $T(\omega) = \frac{V_2}{I_1}$
  - Impedance - $Z(\omega) = \frac{V}{I}$
  - Admittance - $Y(\omega) = I/V$
  - TF Poles/Zeros
    - $H(\omega) = \underbrace{G_0}_{\text{DC gain}}(j\omega)^K \frac{\Pi(1-j\omega\tau_{z,i})}{\Pi(1-j\omega\tau_{p,i})}$ ← zeros ← poles
  - Phasors

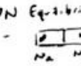
$$Z_C = \frac{1}{j\omega C},\ Z_L = j\omega L,\ Z_R = R$$

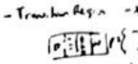
$$R_1 \| R_2 = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

    - Thevenin Equiv -  ← independent - short V, open I - measure R - short out port for I
    - Norton Equiv -  - can also apply current/volt
  - RLC circuit
    - resonance - impedance minimum value
    - voltage gain, but not power gain
    - $Q = \frac{\omega_0 L}{R}$ $\omega_0 = \frac{1}{\sqrt{LC}}$
    - high Q good for super selective filter
  - Bode Plot
    - just add graphs



- Power
  - $P_{avg} = \frac{|I||V|}{2}\underbrace{\cos(\theta_i - \theta_v)}_{PF}$ // nonlinear
- Input Impedance - ~~[illegible]~~ impedance to ground from input
- Output Impedance - impedance to ground from output
- Maxwell's Divergence Eq: $\frac{dE}{dx} = \frac{\rho(x)}{\epsilon_s}$
- Gauss Law $\oint E \cdot dS = \frac{Q_{enc}}{\epsilon_0}$
- $E = -\frac{dV}{dx}$
- 1D Poisson Eqn $\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\epsilon}$

- Intro to Semiconductors
  - Ohm's Law (cons of conversion) V ∝ I
  - $J = \sigma E = \frac{I}{A} = \sigma\frac{V}{L}$; $V = \frac{L}{\sigma A}I = IR$
  - current by + and - charge carriers
  - $\sigma = q^2\left(\frac{N^+\tau^+}{M^+} + \frac{N^-\tau^-}{M^-}\right)$ (ideal gas model)
  - free carriers created w/ and without thermal energy & impurities
  - valence and conduction band (band gap)
  - Free e^- if enough thermal energy (> band gap)
  - Holes form where e^- leave
  - Recombination - e^- + hole (p^+)
  - Thermal Equilibrium
    - generation = recombination
    - $n_0 = p_0$ $np = n_i^2$ Law of Mass Action
    - $n_i \approx 10^{10}cm^{-3} = 10^{16}m^{-3}$
    - $n_i \propto e^{-E_g/kT}$ // $E_g \triangleq$ band gap
  - Doping
    - Group V (Phosphor) (N-type)
    - extra e^-
    - $\rho = -qn_0 + qp_0 + qN_d = 0$ // neutral
    - if $N_d \gg n_i$, $n_0 = N_d$ (N-type)
    - if $N_a \gg n_i$, $p_0 = N_a$ (P-type)
    - Compensation - if $|N_d - N_a| \gg n_i$,
      $n_0 = N_d - N_a$ if $N_d > N_a$
      $p_0 = N_a - N_d$ if $N_a > N_d$
  - Drift Current
    - proportional to applied field E
  - Diffusion Current
    - $J = -qF = qV_{th}\lambda\frac{dn}{dx}$ ← concentration gradient
  - Einstein Relation
    - $D_n = \frac{kT}{q}\mu_n$ ↑ diffusion coeff ↑ mobility
  - Total Current
    - $J = J_{drift} + J_{diff} = q\mu_n nE + qD_n\frac{dn}{dx}$
- IC Stuff
  - IC resistor - poly film or in substrate
    - lots of parasitic capacitance
    - $R = \frac{L}{W}\frac{1}{\sigma t} = R_\square\left(\frac{L}{W}\right)$
  - Boundary stuff - E discontinuous due to polarization
  - IC Cap
    - $C = \frac{A\epsilon}{d}$, $Q = CV$
    - nonlinear cap $Q = f(V_s)$
      - small signal $Q = f(V_0) + \overbrace{\frac{df(V_0)}{dV_0}}^{C} \cdot V_s$
- PN Equilibrium
  -  $V_{th} = \frac{kT}{q} \approx 26mV$
  - $\phi_n = V_{th}\ln\left(\frac{n}{n_i}\right)$ $\phi_p = -V_{th}\ln\left(\frac{p}{n_i}\right)$
  - Transition Region $-x_{p0}$ to x_{n0}

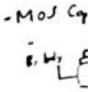


  - Depletion Approximation - 0 free carriers
    - E = 0 outside region
  - $x_{n0} = \sqrt{\frac{2\epsilon_s\phi_{bi}}{qN_d} \cdot \frac{N_a}{N_a + N_d}}$ $x_{p0} = \sqrt{\frac{2\epsilon_s\phi_{bi}}{qN_a} \cdot \frac{N_d}{N_a + N_d}}$

$$X_{d0} = \sqrt{\frac{2\epsilon_s\phi_{bi}}{q}\cdot\left(\frac{1}{N_a} + \frac{1}{N_d}\right)} \quad \phi_{bi} = V_{th}\ln\left(\frac{N_d N_a}{n_i^2}\right)$$

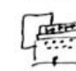
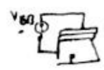
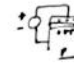
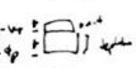
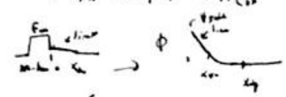
  - not a battery b/c contact potential
  - Reverse Bias $V_D < 0$ ~~[illegible]~~ $\phi_{bi} \Rightarrow \phi_{bi} - V_D$, scale $\sqrt{1 - \frac{V_D}{\phi_{bi}}}$
  - $Q_j = -qN_a x_{p0}\sqrt{1 - \frac{V_D}{\phi_{bi}}}$
  - $C_j = \frac{qN_a x_{p0}}{2\phi_{bi}\sqrt{1 - \frac{V_D}{\phi_{bi}}}} = \frac{C_{j0}}{\sqrt{1 - \frac{V_D}{\phi_{bi}}}}$, $C_{j0} = \frac{\epsilon_s}{X_{d0}}$

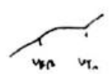
- PN Current
  - $I_d = I_s\left(e^{\frac{qV_d}{kT}} - 1\right)$
  - $\frac{\partial\Delta p}{\partial t} = -C_p N_T \Delta p$ ← $\Delta p = p - p_0$ concentration above equilibrium
    $= -\frac{\Delta p}{\tau_p}$ ← minority carrier lifetime
  - $\frac{dJ_p}{dx} = q\frac{\Delta p}{\tau_p}$ // if current changes, accumulation of charge
  - $J_p = qD_p\frac{d\Delta p}{dx}$ (diffusion current from before)
  - $\frac{d^2\Delta p}{dx^2} = \frac{\Delta p}{D_p\tau_p} = \frac{\Delta p}{L_p^2}$ $L_p = \sqrt{D_p\tau_p}$ (diffusion length)
  - $i_d \approx \frac{qV_d}{kT} = g_d v_d$ // small signal model, forward bias
- MOS Capacitor



at $V_{GB} = 0V$ $\phi_p = -\frac{kT}{q}\ln\left(\frac{N_a}{n_i}\right)$

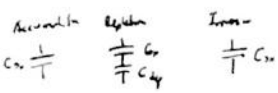
$$\phi_{poly,n^+} = \frac{kT}{q}\ln\left(\frac{N_{d,poly}}{n_i}\right)$$

  -  $V_{GB} = 0$ $V_{FB} = -(\phi_{n^+} - \phi_p)$
  -  $V_{GB} = V_{FB}$ (flat-band voltage) $Q = 0$
  - Accumulation $V_{GB} < V_{FB}$, ‖ plate  $V_{FB} < 0$ $Q_{GB} = C_{ox}(V_{GB} - V_{FB})$
  - Depletion ~~[illegible]~~ $V_{GB} > V_{FB}$  } Depletion from +
  - Inversion $V_{GB} = V_T$
    p-type → n-type
    $V_{GB} > V_T$  inversion p → n
    - $n_s = n_i e^{\frac{q\phi_s}{kT}} = N_a$ $\phi_s = -\phi_p$ 
  - $V_{TH} = V_{FB} - 2\phi_p + \frac{1}{C_{ox}}\sqrt{2q\epsilon_s N_a(-2\phi_p)}$
    $= V_{FB} - 2\phi_p + Q_{dep}/C_{ox}$
- E field 

Q  C 

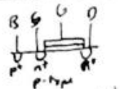
  - $C_{acc} = C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$ ↑ accumulation

Accumulation Depletion Inversion



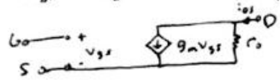
Matthew Tran
EE105
Final

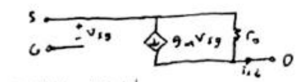
- MOSFET
  - NMOS - PMOS all reversed



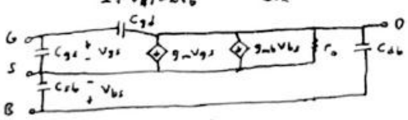
  - Large Signal
    - Cutoff $V_{GS} < V_T$
      - accumulation or depletion
      - no current flow
      - S, B usually connected
    - Inversion $V_{GS} > V_T$, $V_{DS} > 0$
      - charge carriers flow from n^+ regions, fast
      - conducts
    - Triode Region - quadratic-ish, nonlinear

    - Saturation - like current source
  - $I_{DS} = \frac{W}{L}\mu_n C_{ox}(V_{GS}-V_T)V_{DS}$ // linear region $V_{DS} \ll V_{GS}-V_T$
  - $I_{DS} = \frac{W}{L}\mu_n C_{ox}(V_{GS}-V_T-\frac{V_{DS}}{2})V_{DS}$ // band, triode $V_{DS} < V_{GS}-V_T$
  - $I_{DS,SAT} = \frac{\frac{W}{L}\mu_n C_{ox}}{2}(V_{GS}-V_T)^2$ // saturation, $V_{GS} > V_T$, $V_{DS} > V_{GS}-V_T = V_{DS,SAT}$
    - independent of V_{DS}
  - actually $\frac{W}{L}\frac{\mu_n C_{ox}}{2}(V_{GS}-V_T)^2(1+\lambda V_{DS})$
    - channel length modulation
- Amplifier Design
  - bias V_{GS} so midrail, output $< V_{DD}$ and $> V_{GS}-V_T$ ← saturation
  - Small Signal Model
    - $g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_T) = \frac{2 I_{DS}}{V_{GS}-V_T}$
    - $r_o = \left(\frac{\partial I_D}{\partial V_{DS}}\right)^{-1} = \frac{1}{\frac{\mu_n C_{ox}}{2}\cdot\frac{W}{L}(V_{GS}-V_T)^2\lambda} \approx \frac{1}{\lambda I_{DS}}$

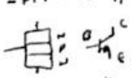




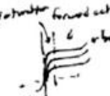
  - Complete Small Signal Model
    - capacitance, various
    - Back gate - body effect — "source" = rail
      - $V_T = V_{T0} + \frac{C_{dep}}{C_{ox}} V_{SB}$
      - $V_T = V_{T0} + \gamma\left(\sqrt{V_{SB}-2\phi_b} - \sqrt{-2\phi_b}\right)$
        - $\gamma = \frac{\sqrt{2q\varepsilon_s N_a}}{C_{ox}}$
      - $g_{mb} = \frac{\gamma g_m}{2\sqrt{-V_{BS}-2\phi_b}} = g_m\frac{C_{dep}}{C_{ox}}$



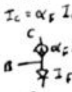
    - PMOS is flipped, see above
- BJT



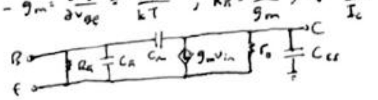
  - E = emitter tied to rail
  - conducts b/c base super thin so minority carrier diffusion crosses
  - $I_C \gg I_B$ (?), $I_C \approx I_E$, $I_C \approx I_S e^{\frac{qV_{BE}}{kT}}$
  - $I_C = \beta_F I_B$
    - ↑ near always



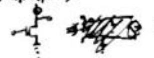
  - Ebers-Moll Model - $I_C = \alpha_F I_{ES}(e^{qV_{BE}/kT}-1)(1+\frac{V_{CE}}{V_A})$
    - good for DC



  - simplified - assume 0.7V, $I_C = \beta I_B$ but in saturation, low voltage
  - Small Signal Model (hybrid-π) $\left(\frac{\partial I_B}{\partial v_{BE}}\right)^{-1}$ $\left(\frac{\partial I_C}{\partial v_{CE}}\right)^{-1}$
    - $g_m = \frac{\partial I_C}{\partial v_{BE}} = \frac{qI_C}{kT}$; $R_\pi = \frac{\beta_F}{g_m}$; $r_o = \frac{V_A}{I_C}$



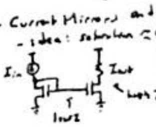
- Single Stage Amplifier
  - Assume unilateral - input not affected by output, but not the other way
  - 4 types defined
    - V→V - ↑R_{in}, ↓R_{out}
    - C→C - ↓R_{in}, ↑R_{out}
    - V→C - ↑R_{in}, ↑R_{out} // transconductance
    - C→V - ↓R_{in}, ↓R_{out} // transresistance
  - Input impedance $Z_{in} = \frac{V_x}{i_x}\Big|_{Z_S\text{ removed},\ Z_L\text{ attached}}$
  - Output impedance $Z_{out} = \frac{V_x}{i_x}\Big|_{Z_L\text{ removed},\ Z_S\text{ attached}}$
  - 3 amp types, CG, CS, CD
  - analysis
    - small signal - large cap = short, small cap = open
    - diff amp types good for diff use cases
  - CS



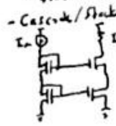
good transconductance
  - CG

current buffer
  - CD

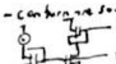
voltage buffer
- Current Mirrors and Biasing
  - idea: saturation ≈ current source



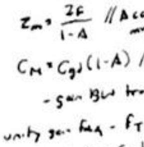
$\frac{W}{L}$ determines scaling
key: saturation
  - Source degeneration - add R_s to all sources
    - boost output impedance
  - Cascode/Stacked Current Source



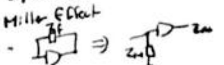
  - less variation
  - but less swing
  - can turn one source into multiple source and sink



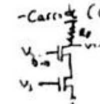
  - $I_{ref} = \frac{V_{DD}-V_{GS,1}}{R_{ref}}$, $V_{GS,1} = V_T + \sqrt{\frac{2I_{ref}}{\mu C_{ox}\frac{W}{L}}}$ ← solve to get I_{ref}
  -



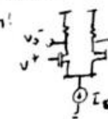
independent of temp and voltage supply
- Frequency Response of Amplifiers
  - 2 poles, LPF and HPF
  - Miller Effect



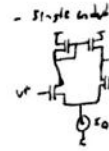
    $Z_m = \frac{Z_F}{1-A}$ // A can change to the f set by circuit
    $C_M = C_{gd}(1-A)$ // capacitor multiplied
      - gain BW tradeoff!
  - unity gain freq - f_T
  - Open Circuit Time Constants (OCTC)
    - assume no zeros and dominant pole ($\omega_1 \ll$ others)
    - $\omega \approx \frac{1}{\Sigma RC}$ // dominated at $\frac{1}{\text{sum of all }\tau}$
      - open each C, measure resistance across
    - usually add high τ node to simplify
- Multi-Stage Amp
  - adjust bottlenecks, gain, BW, etc
  - Cascode (CS + CG)



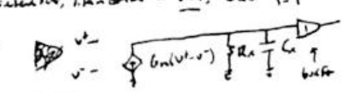
    - lower Miller effect
    - lower swing
    - high output impedance
  - Tech gold tables
- Differential Amplifier
  - Common Mode Noise! ✕
  - need symmetry!



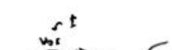
- CMRR (Common Mode Rejection Ratio)
  - how? $CMRR = \frac{|A_{dm}|}{|A_{cm}|}$ // ~70dB-120dB for matched
- Single ended Diff amp



  - double the gain!
  - better CMRR
- Opamp Feedback and Frequency Response
  - $G_{total} = \frac{G}{1+GF} \approx \frac{1}{F}$ as $G \to \infty$
    $= \frac{G}{1+T}$, $T = GF$ } negative feedback
  - noise rejection $\frac{1}{1+T}$
  - positive feedback, like SRAM latches - $\underline{T<1}$ ← important, $G_{total} = \frac{G}{1-T}$
  - opamp model



    - simplified thing
    - Operational Transconductance Amplifier (OTA)
      - remove buffer, = ↗
      - internally common OTA + buffer
  - $\omega_u = GBW$ // unity gain, gain bandwidth tradeoff
  - stability, need < 180° phase lag, 60° margin better
    - small
- Non-ideal Opamp
  - offset voltage



→ shifts TF
  - Input Bias/Offset Current
    - $I_{OS} = |I_{B_1} - I_{B_2}|$ model as current sources into inputs
  - Output Saturation
    - voltage & current limit
  - Slew Rate $SR = \frac{dV_o}{dt}\Big|_{maximum}$
    - nonlinear, diff from BW ← change to gain
    - normally $v_o(t) = V_i(1-e^{-t\omega_u})$ for step input
      - $\frac{dv}{dt}\max = V_i\omega_u < SR$
    - usually b/c internal cap and current limit
      $SR = \pm\frac{I_{max}}{C_x}$
  - Full Power BW - ω when SR happens $\omega_{max} V_{i,max} = SR$
  - Opamp Distortion - nonlinearity, harmonics
  - Opamp Noise - thermal fluctuations, etc, white noise } usually v small